Weak Matrix Elements without Quark Masses on the Lattice

L. Giusti[a*]

[a]Department of Physics, Boston University Boston, MA 02215 USA.

We introduce a new parameterization of four-fermion matrix elements which does not involve quark masses and thus allows a reduction of systematic uncertainties in physical amplitudes. As a result the apparent quadratic dependence of ϵ'/ϵ on $m_s(\mu)$ is removed. To simplify the matching between lattice and continuum renormalization schemes, we express our results in terms of Renormalization Group Invariant B-parameters which are renormalization-scheme and scale independent. As an application of our proposal, matrix elements of $\Delta I = 3/2$ and SUSY $\Delta F = 2$ ($F = S, C, B$) four-fermion operators have been computed.

1. Introduction

Since the original proposals of using lattice QCD to study hadronic weak decays [1–3], substantial theoretical and numerical progress has been made: the main theoretical aspects of the renormalization of composite four-fermion operators are well understood [4–6]; the calculation of K^0–$\bar{K}^0$ mixing, relevant to the prediction of the CP-violation parameter ϵ, has reached a level of accuracy which is unpaired by any other approach [7]; increasing precision has been gained in the determination of the electro-weak penguin amplitudes necessary to the prediction of the CP-violation parameter ϵ'/ϵ [8–10]; finally matrix elements of $\Delta S = 2$ operators which are relevant to study FCNC effects in SUSY models have been computed [9,10]. Methods and symbols used in this talk and all the results we report are fully described in [9,10].

2. Matrix elements without quark masses

The analysis of $K^0 - \bar{K}^0$ mixing with the most general $\Delta S = 2$ effective Hamiltonian requires the knowledge of the matrix elements $\langle \bar{K}^0|O_i|K^0\rangle$ of the parity conserving parts of the following operators

$$\begin{aligned} O_1 &= \bar{s}^\alpha\gamma_\mu(1-\gamma_5)d^\alpha\ \bar{s}^\beta\gamma_\mu(1-\gamma_5)d^\beta, \\ O_2 &= \bar{s}^\alpha(1-\gamma_5)d^\alpha\ \bar{s}^\beta(1-\gamma_5)d^\beta, \\ O_3 &= \bar{s}^\alpha(1-\gamma_5)d^\beta\ \bar{s}^\beta(1-\gamma_5)d^\alpha, \\ O_4 &= \bar{s}^\alpha(1-\gamma_5)d^\alpha\ \bar{s}^\beta(1+\gamma_5)d^\beta, \\ O_5 &= \bar{s}^\alpha(1-\gamma_5)d^\beta\ \bar{s}^\beta(1+\gamma_5)d^\alpha. \end{aligned} \tag{1}$$

On the lattice, matrix elements of weak four-fermion operators are computed from first principles. But, following the common lore, they are usually given in terms of the so-called B-parameters which measure the deviation of their values from those obtained in the Vacuum Saturation Approximation (VSA). For operators in (1), the B-parameters are usually defined as

$$\begin{aligned} \langle\bar{K}^0|O_1(\mu)|K^0\rangle &= \frac{8}{3}M_K^2 f_K^2 B_1(\mu)\,, \\ \langle\bar{K}^0|O_i(\mu)|K^0\rangle &= \frac{C_i}{3}\left(\frac{M_K^2 f_K}{m_s(\mu)+m_d(\mu)}\right)^2 B_i(\mu)\,, \end{aligned} \tag{2}$$

where $C_i = -5, 1, 6, 2$ for $(i = 2, \ldots, 5)$. In (2), $\langle\bar{K}^0|O_1|K^0\rangle$ is parameterized in terms of well-known experimental quantities and $B_1(\mu)$ ($B_K(\mu) \equiv B_1(\mu)$). On the contrary, $\langle\bar{K}^0|O_i|K^0\rangle$ $(i = 2, \ldots, 5)$ depend quadratically on the quark masses in (2), while they are expected to remain finite in the chiral limit and depend only linearly on the quark masses. Contrary to f_K, M_K, etc., quark masses can not be directly measured by experiments and the present accuracy in their determination is still rather poor. Therefore, whereas for O_1 we introduce B_K as an alias of the matrix element, by using (2) we replace each of the SUSY matrix elements with 2 unknown quantities, i.e. the B-parameter and m_s+m_d. To overcome these

*Invited talk at the QCD Euroconference 99. Work done in collaboration with A. Donini, V. Gimenéz and G. Martinelli. This research was supported in part under DOE grant DE-FG02-91ER40676.

Table 1
Matrix elements in GeV^4 at the renormalization scale $\mu = 2$ GeV in the RI scheme. In the first two columns the results obtained with the new parameterization are given, while in the last two columns we show the results obtained in ref. [9] with the "conventional" parameterization on the same set of data.

$\langle O_i \rangle$	New $\beta = 6.0$ this work	New $\beta = 6.2$ this work	Old $\beta = 6.0$ [9]	Old $\beta = 6.2$ [9]
$\langle O_1 \rangle$	0.012(2)	0.011(3)	0.012(2)	0.011(3)
B_1	0.70(15)	0.68(21)	0.70(15)	0.68(21)
$\langle O_2 \rangle$	-0.079(10)	-0.074(8)	-0.073(15)	-0.073(15)
B_2	0.72(9)	0.67(7)	0.66(3)	0.66(4)
$\langle O_3 \rangle$	0.027(2)	0.021(3)	0.025(5)	0.022(5)
B_3	1.21(10)	0.95(15)	1.12(7)	0.98(12)
$\langle O_4 \rangle$	0.151(7)	0.133(12)	0.139(28)	0.133(28)
B_4	1.15(5)	1.00(9)	1.05(3)	1.01(6)
$\langle O_5 \rangle$	0.039(3)	0.029(5)	0.035(7)	0.029(7)
B_5	0.88(6)	0.66(11)	0.79(6)	0.67(10)
$\langle O_7^{3/2} \rangle$	0.019(2)	0.011(3)	0.020(5)	0.014(5)
$B_7^{3/2}$	0.65(5)	0.38(11)	0.68(7)	0.46(13)
$\langle O_8^{3/2} \rangle$	0.082(4)	0.068(8)	0.092(19)	0.087(19)
$B_8^{3/2}$	0.92(5)	0.77(9)	1.04(4)	0.98(8)

problems, we propose the following new parameterization of $\Delta S = 2$ operators

$$\begin{aligned} \langle \bar{K}^0 | O_1(\mu) | K^0 \rangle &= \frac{8}{3} M_K^2 f_K^2 B_1(\mu), \\ \langle \bar{K}^0 | O_i(\mu) | K^0 \rangle &= M_{K^*}^2 f_K^2 \tilde{B}_i(\mu). \end{aligned} \tag{3}$$

The $\tilde{B}_i(\mu)$ parameters are still dimensionless quantities and can be computed on the lattice by studying appropriate ratios of three- and two point functions [10]. By simply using them, we have eliminated any fictitious reference to the quark masses, hence reducing the systematic errors on the corresponding physical amplitudes. An alternative parameterization which has not been used in our numerical analysis, but may be very useful in the future, can be found in [10].

Table 2
RGI Matrix elements in GeV^4 computed as in Eq. (11) with $\alpha_s^{n_f=4}$.

$\langle O_i^{RGI} \rangle$	$\beta = 6.0$	$\beta = 6.2$
$\langle O_1^{RGI} \rangle$	0.017(3)	0.016(4)
$\langle O_2^{RGI} \rangle$	-0.051(7)	-0.048(6)
$\langle O_3^{RGI} \rangle$	0.005(7)	-0.004(7)
$\langle O_4^{RGI} \rangle$	0.072(3)	0.063(6)
$\langle O_5^{RGI} \rangle$	0.043(3)	0.032(5)

The VSA and B-parameters are also used for matrix elements of operators which enter the $\Delta S = 1$ effective Hamiltonian. Notice that this "conventional" parameterization is the only responsible for the apparent quadratic dependence of ϵ'/ϵ on the quark masses. This introduces a redundant source of systematic error which can be avoided by parameterizing the matrix elements in terms of measured experimental quantities and therefore *a better determination of the strange quark mass $m_s(\mu)$ will not improve our theoretical knowledge of ϵ'/ϵ*. In this work we have computed the matrix elements $\langle \pi | O_i^{3/2} | K \rangle$ of the four fermion operators $O_i^{3/2}$ $(i = 7, 8, 9)$ which contribute to the $\Delta I = 3/2$ sector of ϵ'/ϵ. In fact in the chiral limit $\langle \pi\pi | O_i^{3/2} | K \rangle$ can be obtained, using soft pion theorems, from $\langle \pi^+ | O_i^{3/2} | K^+ \rangle$. For degenerate quark masses, $m_s = m_d = m$, and in the chiral limit, we find

$$\begin{aligned} \lim_{m\to 0} \langle \pi^+ | O_7^{3/2} | K^+ \rangle &= -M_\rho^2 f_\pi^2 \lim_{m\to 0} \tilde{B}_5(\mu) \\ \lim_{m\to 0} \langle \pi^+ | O_8^{3/2} | K^+ \rangle &= -M_\rho^2 f_\pi^2 \lim_{m\to 0} \tilde{B}_4(\mu) \\ \lim_{m\to 0} \langle \pi^+ | O_9^{3/2} | K^+ \rangle &= \frac{8}{3} M_\pi^2 f_\pi^2 \lim_{m\to 0} B_1(\mu) \,. \end{aligned}$$

In the limit $m_s = m_d$ complicated subtractions of lower dimensional operators are avoided for $\Delta I = 3/2$ operators. This is not the case for $\Delta I = 1/2$ operators which enter the determination of ϵ'/ϵ : in this case the mixing with lower dimensional operators makes the computation much more involved. A reliable lattice estimate of these matrix elements is still missing but encouraging preliminary results with domain-wall fermions have been presented in ref. [11].

3. Renormalization Group Invariant Operators

Physical amplitudes can be written as

$$\langle F | \mathcal{H}_{eff} | I \rangle = \langle F | \vec{O}(\mu) | I \rangle \cdot \vec{C}(\mu) \,, \tag{4}$$

where $\vec{O}(\mu) \equiv (O_1(\mu), \ldots, O_N(\mu))$ is the operator basis (for example the basis defined in (1) for the $\Delta S = 2$ effective Hamiltonian) and $\vec{C}(\mu)$ the corresponding Wilson coefficients represented as a column vector. $\vec{C}(\mu)$ is expressed in terms of its counter-part, computed at a large scale M, through the renormalization-group evolution matrix $\hat{W}[\mu, M]$

$$\vec{C}(\mu) = \hat{W}[\mu, M]\vec{C}(M)\,, \tag{5}$$

where the initial conditions $\vec{C}(M)$, are obtained by perturbative matching of the full theory to the effective one at the scale M where all the heavy particles have been removed. $\hat{W}[\mu, M]$ can be written as (see for example [12])

$$\hat{W}[\mu, M] = \hat{M}[\mu]\hat{U}[\mu, M]\hat{M}^{-1}[M]\,, \tag{6}$$

where $\hat{U} = (\alpha_s(M)/\alpha_s(\mu))^{(\gamma_O^{(0)T}/2\beta_0)}$ is the leading-order evolution matrix and $M(\mu)$ is a NLO matrix defined in [12] which can be obtained by solving the Renormalization Group Equations (RGE) at the next-to-leading order. The Wilson coefficients $\vec{C}(\mu)$ and the renormalized operators $\vec{O}(\mu)$ are usually defined in a given scheme, at a fixed renormalization scale μ, and they depend on the renormalization scheme and scale in such a way that only H_{eff} is scheme and scale independent. This is a source of confusion in the literature, especially when (perturbative) coefficients and (non-perturbative) matrix elements are computed using different techniques, regularization, schemes and renormalization scales. To simplify the matching procedure, we propose a Renormalization Group Invariant (RGI) definition of Wilson coefficients and composite operators which generalizes what is usually done for B_K and for quark masses [13,14]. We define

$$\hat{w}^{-1}[\mu] \equiv \hat{M}[\mu]\,[\alpha_s(\mu)]^{-\hat{\gamma}_O^{(0)T}/2\beta_0}\,, \tag{7}$$

and using Eqs. (6) and (7) we obtain

$$\hat{W}[\mu, M] = \hat{w}^{-1}[\mu]\hat{w}[M]\,. \tag{8}$$

The effective Hamiltonian (4) can be written as

$$\mathcal{H}_{eff} = \vec{O}^{RGI} \cdot \vec{C}^{RGI}\,, \tag{9}$$

where

$$\vec{C}^{RGI} = \hat{w}[M]\vec{C}(M)\,,\ \vec{O}^{RGI} = \vec{O}(\mu)\cdot\hat{w}^{-1}[\mu]\,. \tag{10}$$

$\vec{C}^{RGI}$ and $\vec{O}^{RGI}$ are scheme and scale independent at the order we are working. Therefore the effective Hamiltonian is splitted in terms which are individually scheme and scale independent. This procedure is generalizable to any effective weak Hamiltonian. The $\tilde{B}$-parameters defined in eqs. (3) satisfy the same RGE as the corresponding operators and the RGI $\tilde{B}$-parameters can be defined as

$$\tilde{B}_i^{RGI} = \sum_j \tilde{B}_j(\mu) w(\mu)^{-1}_{ji}\,. \tag{11}$$

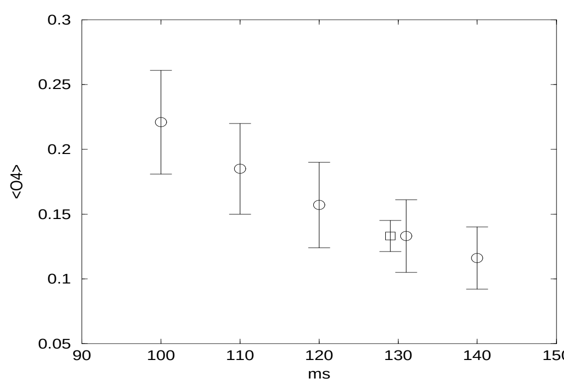

Figure 1. Comparison of $\langle \bar{K}^0|O_4|K^0\rangle$ in GeV^4 computed with the new (square) and the old (circle) parameterization as a function of the strange quark mass used in (2) to obtain the full matrix element from the B parameters.

4. Numerical results

All details concerning the extraction of matrix elements from correlation functions and the computation of the non-perturbative renormalization constants of lattice operators can be found in [6,9,10]. In this talk we report the results obtained in [10]. The simulations have been performed at $\beta = 6.0$ (460 configurations) and 6.2 (200 configurations) with the tree-level Clover action, for several values of the quark masses and

for different meson momenta. The physical volume is approximatively the same on the two lattices. Statistical errors have been estimated with the jacknife method. The main results we have obtained for $\Delta S = 2$ and $\Delta I = 3/2$ matrix elements and their comparison with the results in [9] are reported in Tables 1 and 2. In Figure 1 we

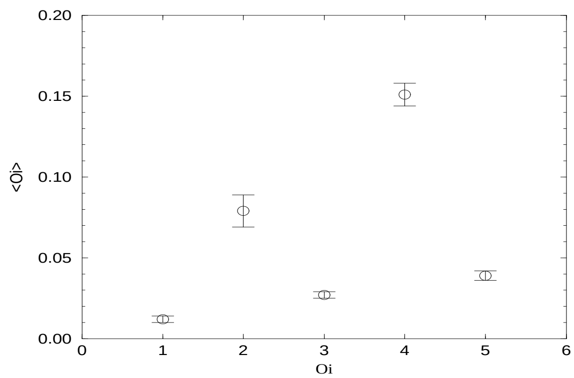

Figure 2. Values of $\langle \bar{K}^0 | O_i | K^0 \rangle$ in GeV^4 computed at $\mu = 2$ GeV in the RI scheme. O_1 corresponds to the Standard Model $\Delta S = 2$ operator.

show the strong dependence of $\langle \bar{K}^0 | O_4 | K^0 \rangle$ on the strange quark mass when the "conventional" parameterization (2) is used, to be compared with the results obtained with the new parameterization. It is also evident that with the same set of data the new parameterization allows to determine the matrix elements with smaller systematic uncertainties. Although we have data at two different values of the lattice spacing, the statistical errors, and the uncertainties in the extraction of the matrix elements, are too large to enable any extrapolation to the continuum limit $a \to 0$: within the precision of our results we cannot study the dependence of $\tilde{B}$-parameters on a. For this reason, we estimate our best values of the B-parameters by averaging the results obtained at the two values of β [10]. Since the results at $\beta = 6.0$ have smaller statistical errors but suffer from larger discretization effects, we do not weight the averages with the quoted statistical errors but take simply the sum of the two values divided by two. As far as the errors are concerned, we take the largest of the two statistical errors. Our best results are reported in Table 3 and are shown in fig. 2. It is interesting to note, as expected from chiral perturbation theory, that matrix elements of $\Delta S = 2$ SUSY operators are enhanced respect to the SM one ($\langle \hat{O}_1 \rangle$) by a factor $2 - 12$ at $\mu = 2$ GeV. Therefore, low energy QCD effects can enhance contributions beyond the Standard Model to ϵ_K [15,16] which, compared with the other SM predictions, becomes a promising observable to detect signals of new physics at low energy. The results for the analogous $\Delta C = 2$ and $\Delta B = 2$ matrix elements are reported in [17].

Table 3
Matrix elements in GeV^4 at $\mu = 2$ GeV in the RI scheme and their RGI values with $\alpha_s^{n_f=4}$.

$\langle O_i \rangle$	RI	RGI
$\langle O_1 \rangle$	0.012(3)	0.017(4)
$\langle O_2 \rangle$	-0.077(10)	0.050(7)
$\langle O_3 \rangle$	0.024(3)	0.001(7)
$\langle O_4 \rangle$	0.142(12)	0.068(6)
$\langle O_5 \rangle$	0.034(5)	0.038(5)

REFERENCES

1. N. Cabibbo *et al.*,Nucl.Phys. B244(1984)381.
2. R.Brower *et al.*,Phys.Rev.Lett. 53(1984)1318.
3. C. Bernard in Argonne 1984 p.85.
4. M.Bochicchio *et al.*Nucl.Phys.B262(1985)331
5. S. Sharpe *et al.*,Nucl.Phys. B286(1987)253.
6. A. Donini *et al.*,Eur.Phys.J. C10(1999)121 and ref. therein.
7. R. Gupta,hep-ph/9801412 and ref. therein. L. Lellouch,hep-ph/9906497 and ref. therein.
8. T.Bhattacharya *et al.*Ph.Rev.D55(1997)4036
9. C. R. Allton et al., Phys. Lett. B453(1999)30.
10. A. Donini et al., in preparation.
11. A. Soni et al., Lattice 99 - Pisa (Italy).
12. M. Ciuchini *et al.*,Nucl.Phys. B523(1998)501.
13. A. Gonzalez Arroyo et al., Phys.Lett.B 117(1982)437; Erratum-ibid. B122(1983)486.
14. S.Capitani *et al.*,Nucl.Phys. B544(1999)669.
15. M. Ciuchini et al., JHEP 10 (1998) 008.
16. L. Giusti et al.,Nucl.Phys. B550(1999)3.
17. L. Conti *et al.*, in preparation.